Torque Analysis of Permanent Magnet Synchronous Motor Used in Automotive Industry

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Abstract— **This paper describes an investigation of torque analysis for permanent magnet synchronous motor are used in automotive industry. It is mathematically proven that the increase of electromagnetic torque in a permanent magnet motor is proportional to the increase of the angle between the stator and rotor flux linkages and therefore the fast torque response can be obtained by adjusting the rotating speed of the stator flux linkage as fast as possible. It is also shown that the zero voltage vectors should not be used and stator flux linkage should be kept moving with respect to the rotor flux linkage all the time. The implementation of torque analysis in the permanent magnet motor is discussed; it is advantageous to have a motor with a high ratio of the rated stator flux linkage to stator voltage. It is seen in result table that by varying the frequency & load, the speed, torque & output power also varying but at a particular frequency range from 33.33Hz to 59Hz the value of torque is constant. Hence constant torque is very useful for Electric power steering in Automotive Industry.**

Keywords— **permanent magnet synchronous motor, stator flux linkage. Rotor flux linkage, electromagnetic torque.**

I. INTRODUCTION

A. Permanent Magnet Synchronous Motor

In a permanent magnet synchronous motor, the dc field winding of the rotor is replaced by a permanent magnet. The advantages are elimination of field copper loss, higher power density, lower rotor inertia and more robust construction of the rotor. The demerits are loss of flexibility of field flux control and possible demagnetization effect. The permanent magnet synchronous motor has higher efficiency than an induction motor, but generally its cost is higher, which makes the life cycle cost of the drive somewhat lower permanent magnet synchronous motor particularly at low power range are widely used in industry. Recently, the interest in their application is growing, particularly up to 100 KW, only reluctance motor are simpler in construction and in assembly procedure then permanent magnet synchronous motor, but reluctance motor generally developed less torque per unit of current and per unit of weight. Therefore, on a basis of power output per unit weight and per unit volume the permanent magnet synchronous motor is superior to all other brushless synchronous motor especially with the commercial feasibility of rare earth magnets. In this motor the magnets are mounted inside the rotor. The motor is connected on load and its speed depends on the stator supply frequency.

II PMSM MODELING

A. Torque Analysis of PMSM

Permanent Magnet synchronous motors (PMSM's) are used in many applications that require rapid torque response and high performance operation. The torque in PMSM's is usually controlled by controlling the armature current based on the fact that the electromagnetic torque is proportional to the armature current. For high performance the current control is normally executed in the rotor dq reference frame that rotates with the synchronous speed. In this frame, the armature inductances and magnet flux linkage are constant if the back electromotive force (EMF) and variation of inductances are sinusoidal. In addition to the influence of the harmonic terms in inductances and back EMF, saturation in flux, and temperature effect on the magnet, the torque response under current control is limited by the time constant of the armature windings. Motor Equation in the stator flux reference frame: The stator flux linkage vector φ_s and rotor (magnet) flux linkage vector φ_f can be drawn in the rotor flux (dq), stator flux (xy) and stationary (dq) reference frames, as in fig.1. The angle between the stator and rotor flux linkages δ is the load angle when the stator resistance is neglected. In the steady state, δ is constant corresponding to a load torque, and both stator and rotor flux rotate at the synchronous speed. In transient operation, δ varies and the stator and rotor flux rotate at different speeds. Since the electrical time constant is normally much smaller than the mechanical time constant, the rotating speed of stator flux, with respect to the rotor flux, can be easily changed. It is shown in this section that the increase of torque can be controlling the change of δ or the rotating speed of the stator flux.

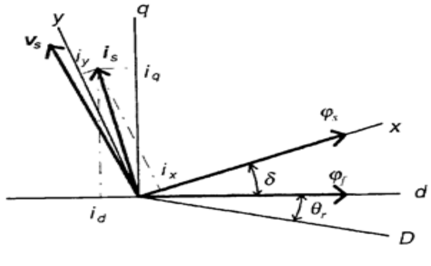

Fig.1The Stator & Rotor Flux Linkages in different reference frames

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The well-known stator flux linkage, voltage, and electromagnetic torque equations in the dq reference frame are as follows:

jd =$L_{d\,d}^{\;i}$ + j_f
jd = $L_{q\,q}^{\;i}$
v_d = $R_{s\,d}^{\;i}$ + pjd - $w_r$jq ————————(A)
v_d = $R_{s\,q}^{\;i}$ + pjq - $w_r$jd
T = 3/2 p ($j_{d\,q}^{\;i}$ - $j_{q\,d}^{\;i}$)

Where j_f, L_d, and L_q are the armature (or stator) back EMF constant and inductances, respectively, when the back EMF and the variation of the stator inductances are sinusoidal. Otherwise, these are the fundamental quantities of these variables. With the transformation in (1) and (2), (33)-(35) can be transformed to the xy reference frame:

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} F_d \\ F_q \end{bmatrix}. \qquad \text{———————(1)}$$

The inverse transformation is

$$\begin{bmatrix} F_d \\ F_q \end{bmatrix} = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}. \qquad \text{———————(2)}$$

Where F represents the voltage, current, and flux linkage.

A) The torque Equation in xy Reference Fram

From fig.1, it can be found that

$$\sin\delta = \frac{\varphi_q}{|\varphi_s|}$$

$$\cos\delta = \frac{\varphi_d}{|\varphi_s|} \qquad \text{———————(3)}$$

Where |j_s| represents the amplitude of the stator flux linkage. Substituting (2) and (3) for current into (A) gives

$$T = \frac{3}{2} p \left[\varphi_d (\imath_x \sin\delta + \imath_y \cos\delta) - \varphi_d (\imath_x \cos\delta - \imath_y \sin\delta)\right]$$

$$= \frac{3}{2} p \left[\imath_x \frac{\varphi_d \varphi_q}{|\varphi_s|} + i_y \frac{\varphi_d^2}{|\varphi_s|} - i_x \frac{\varphi_d \varphi_q}{|\varphi_s|} + i_y \frac{\varphi_d^2}{|\varphi_s|} \right] = \frac{3}{2} p |\varphi_s| i_y$$

———————(4)

Equation (4) means that the torque is directly proportional to the y-axis component of the stator current if the amplitude of the stator flux linkage is constant.

B) The flux linkage Equation in the xy reference frame

Equation (3) can be rewritten into matrix from as follows:

$$\begin{bmatrix} \varphi_d \\ \varphi_q \end{bmatrix} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \varphi_f \\ 0 \end{bmatrix} \qquad \text{———————(5)}$$

Substituting (2) into (5) gives

$$\begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} \varphi_x \\ \varphi \end{bmatrix} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} + \begin{bmatrix} \varphi_f \\ 0 \end{bmatrix}$$

———(6)

Premultiplying (6) with

$$\begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix}^{-1} = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix}$$

———————(7)

gives (8),

$$\begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} = \begin{bmatrix} L_d \cos\delta & L_q \sin\delta \\ -L_d \sin\delta & L_q \cos\delta \end{bmatrix} \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} + \varphi_f \begin{bmatrix} \cos\delta \\ -\sin\delta \end{bmatrix}$$

$$= \begin{bmatrix} L_d \cos^2\delta + L_q \sin^2\delta & -L_d \sin\delta\cos\delta + L_q \sin\delta\cos\delta \\ -L_d \sin\delta\cos\delta + L_q \sin\delta\cos\delta & L_d \sin^2\delta + L_q \cos^2\delta \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} + \varphi_f \begin{bmatrix} \cos\delta \\ -\sin\delta \end{bmatrix}$$

—(8)

1) PMSM'a with Uniform Airgap: for this type of PMSM, $L_d = L_q = L_s$ can be simplified as in (12)

$$\begin{bmatrix} \varphi_x \\ \varphi \end{bmatrix} = \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} + \varphi_f \begin{bmatrix} \cos\delta \\ -\sin\delta \end{bmatrix}$$

————(9)

or

j_x = $L_{s\,x}^{\;i}$ +$j_f$cosd
j_y = $L_{s\,s}^{\;i}$ - j_f sind ————————(10)

j_y is zero since the x- axis is fixed at the stator flux linkage. Then, i_y can be solved from the second equation of (45)

i_y = 1/ L_s x j_f sind ————————(11)

Substituting (11) into torque (4) give

$$T = \frac{3}{2}\frac{1}{L_s} p |\varphi_s| \varphi_f | \sin\delta = \frac{3}{2}\frac{1}{L_s} p |\varphi_s| \varphi_f | \sin\delta t$$

————————(12)

Where d is the angular velocity of the stator flux linkage relative to magnet flux linkage.

Equation (12) implies that the torque increases with the increase in d if the amplitude of the stator flux linkage is kept constant and d is controlled within the range of -p/2 -p/2.The maximum torque occurs when d is p/2. d is considered to be a step change corresponding to a change of voltage vector. Then, the derivative of (12) becomes

$$\left.\frac{dT}{dt}\right|_{t=0} = \frac{3}{2} p \frac{|\varphi_s| \varphi_f}{L_s} \delta \cos\delta \qquad \text{———————(13)}$$

The right-hand side of (13) is always positive if dis positive if d is within the range of -p/2-p2. This equation implies that the increase of torque is proportional to the increase of the angle d, which is the angle between the stator and magnet flux linkage. In other words, the stator flux linkage should be controlled in such a way that the amplitude is kept constant and the rotating speed is controlled as fast as possible to obtain the maximum change in actual torque.

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2) PMSMs with pole saliency: for a PMSM with pole saliency, that is, L--$_d$ ¹L$_q$, the torque equation in terms of stator flux linkage and angle d can be obtained by solving i$_x$ from (44), with j$_y$ = 0:

$$i_x = \frac{2\varphi_f \sin\delta - [(L_d + L_q) + (L_d - L_q)\cos 2\delta}{(L_q - L_d)\sin 2\delta} i_y$$

(14)

Substituting (14) into the first equation in (8), one obtains

$$i_y = \frac{1}{2L_d L_q}[2\varphi_f L_q \sin\delta - |\varphi_s|(L_q - L_d)\sin 2\delta$$

(15)

Then, the torque equation is as follows:

$$T = \frac{3p|\varphi_s|}{4L_d L_q}[2\varphi_f L_q \sin\delta - |\varphi_s|(L_q - L_d)\sin 2\delta]$$

(16)

Equation (16) consists of two terms. The first is the excitation torque, which is produced by the permanent magnet flux, and the second term is the reluctance torque. For each stator flux linkage, there exists the maximum in this equation. It will not be discussed how to control the amplitude of stator flux linkage and load angle to get maximum torque. However, it is necessary to discuss the relationship between the amplitude of stator flux linkage and the derivative of the torque. Fig. 2 shows the torque -d characteristics when the amplitude of stator flux linkage is at j$_{f:}$ Therefore, for a PMSM with pole saliency, the amplitude of the stator flux linkage should be changer, with the change of actual torque even for constant torque operation. The derivative of torque in (17) is as shown in (53), with constant stator flux and dd/dt:

$$\frac{dT}{dt} = \frac{3p|\varphi_s|}{4L_d L_q}[2\varphi_f L_q \delta\cos\delta - 2|\varphi_s|(L_q - L_d)\delta\cos 2\delta]$$

(17)

At t = 0:

$$\left.\frac{dT}{dt}\right|_{t=0} = \frac{3p|\varphi_s|}{2L_d L_q}[\varphi_f L_q \delta - |\varphi_s|(L_q - L_d)\delta]$$

(18)

The condition for dT/dt for positive dd/dt is

$$|\varphi_s| < \frac{L_q}{L_q - L_d}\varphi_f \qquad (19)$$

The Primary voltage vector v$_s$ is defined by the following equation

$$v_s = \frac{2}{3}(v_a + v_b e^{i(2/3)\pi} + v_c e^{(4/3)\pi} \qquad (20)$$

The amplitude of the stator flux linkage should be chosen according to (20) if fast dynamic response is desired. Otherwise, the should be varied with the change of actual torque if the linearity is more important. It should also be kept in mind that for the same torque, a higher stator current is needed when the amplitude of the stator flux linkage is lower.

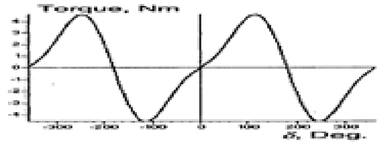

Fig. 2 Torque with respective d: |j$_s$| = j$_f$

III PROPOSED METHOD

Permanent magnet synchronous motor is used here with three stator windings for the motor operation. Three supply voltage are obtained with the help of three phase MOSFET bridge inverters. MOSFET bridges are fed with fixed dc voltage which is obtained by rectifying ac voltage available from ac mains with the help of diode bridge. Shunt capacitor filter is used for filtering purpose. Operation of the MOSFET bridge is controlled by the control circuit. Gating pulses required to turn the MOSFET On are obtained from the control circuit. By controlling the frequency of the gating pulses frequency of the output from MOSFET bridge is controlled. Control circuit consists of clock generator counter and EPROM. First data required to generate gating pulses is calculated and is stored in EPROM. This data is outputted at the output of the EPROM by generating the address of the memory location with the help of 4 bit binary ripple counter. Clock input required for the operation of the counter is generated using IC 555 in a stable mode. Frequency of the gating signals coming out of EPROM is dependent on the frequency with which addressing is done which is turn dependent on the clock frequency. Thus by varying the clock frequency of gating signal is varied. If frequency of gating signal is varied, then the MOSFET bridge output frequency is also varied. Thus we obtain variable frequency output. Gating signal outputted by EPROM can not be directly applied to MOSFET Bridge as they are very weak. So isolator and driver circuit is used. Necessary isolation of low power control circuit from high power bridge circuit is obtained by using opt isolator. In interior or buried magnet synchronous motor (IPM), the magnets are mounted inside the rotor. The motor is connected on load and its speed depends on the stator supply frequency.

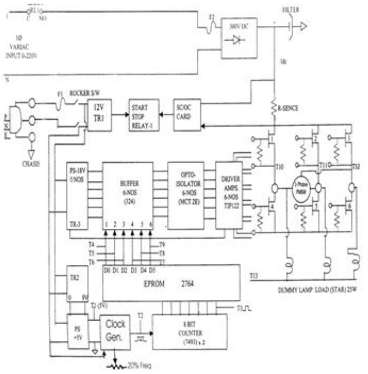

Fig.3 Block Diagram of PMSM Drive for Torque Analysis

IV.RESULT TABLE

TABLE I.

Sr. No.	Load (gm)	Frequency (Hz)	Speed (rpm)	Torque (N-m)	Output Power (W)
1	500	33.3	1010	0.14715	15.55
2	1000	33.3	1010	0.2943	31.12
3	1500	33.3	1010	0.4414	46.68
4	2000	33.3	1010	0.5886	62.19
5	2500	33.3	1010	0.7357	77.81
6	3000	33.3	1010	0.8829	93.38

TABLE II.

Sr. No.	Load (gm)	Frequency (Hz)	Speed (rpm)	Torque (N-m)	Output Power (W)
1	500	40.0	1226	0.14715	18.93
2	1000	40.0	1226	0.2943	37.78
3	1500	40.0	1226	0.4414	56.66
4	2000	40.0	1226	0.5886	75.56
5	2500	40.0	1226	0.7357	94.45
6	3000	40.0	1226	0.8829	113.35

TABLE III.

Sr. No.	Load (gm)	Frequency (Hz)	Speed (rpm)	Torque (N-m)	Output Power (W)
1	500	45.4	1380	0.14715	21.31
2	1000	45.4	1380	0.2943	42.53
3	1500	45.4	1380	0.4414	63.78
4	2000	45.4	1380	0.5886	85.06
5	2500	45.4	1380	0.7357	106.31
6	3000	45.4	1380	0.8829	127.59

TABLE IV.

Sr. No.	Load (gm)	Frequency (Hz)	Speed (rpm)	Torque (N-m)	Output Power (W)
1	500	50	1520	0.14715	23.47
2	1000	50	1520	0.2943	46.84
3	1500	50	1520	0.4414	70.25
4	2000	50	1520	0.5886	93.68
5	2500	50	1520	0.7357	117.1
6	3000	50	1520	0.8829	140.5

TABLE V.

Sr. No.	Load (gm)	Frequency (Hz)	Speed (rpm)	Torque (N-m)	Output Power (W)
1	500	55.5	1682	0.14715	25.98
2	1000	55.5	1682	0.2943	51.83
3	1500	55.5	1682	0.4414	77.74
4	2000	55.5	1682	0.5886	103.67
5	2500	55.5	1682	0.7357	129.58
6	3000	55.5	1682	0.8829	155.51

TABLE VI.

Sr. No.	Load (gm)	Frequency (Hz)	Speed (rpm)	Torque (N-m)	Output Power (W)
1	500	59	1790	0.14715	27.64
2	1000	59	1790	0.2943	55.16
3	1500	59	1790	0.4414	82.73
4	2000	59	1790	0.5886	110.33
5	2500	59	1790	0.7357	137.90
6	3000	59	1790	0.8829	165.49

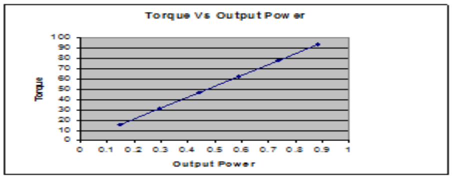

Fig. 4 Torque Vs Output Power at frequency 33.3 Hz

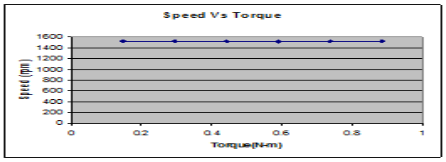

Fig. 5 Speed Vs Torque at Constant Frequency = 50 Hz

CONCLUSIONS

This paper presented the description and result of the Torque analysis of PMSM used in Automotive Industry. It is mathematically proven that the increase of electromagnetic torque in a permanent magnet motor is proportional to the increase of the angle between the stator and rotor flux linkages and therefore, the fast torque response can be obtained by adjusting the rotating speed of the stator flux linkage as fast as possible. It is seen in result table and waveform. And it is found that the motor with a high ratio of the rated stator flux linkage-to-stator voltage is required for controlling the flux linkage.

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